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PRIME MINISTER

COMPOUND INTEREST

David has mentioned to you his neat way of calculating how many years it takes for a sum to double at a particular rate of compound interest. You simply divide 69 (not 67) by the interest rate. You might like to know why this works:

Let x be the rate of interest, and y the number of years for the sum to double.

Then $(1 + x)^y = 2$

Taking logs $y \log (1 + x) = \log 2$

In natural logs, the first degree approximation $\log (1 + x)$ is simply x, when x is small. (The next term is a function of x^2. So when x is small, that term and subsequent ones are very small.) The fact that the first order approximation of $\log (1 + x)$ for small x is so good is the key to the accuracy and simplicity of David's method.

So $y \log (1 + x) = \log 2$

$\Rightarrow$ $yx = \log 2$ when x is small.

And $\log 2 = 0.69$

$\Rightarrow$ $y = \frac{0.69}{x}$ or where x is expressed as a percentage $\frac{69}{x}$

This approximation works surprisingly well. The following table compares the actual time taken to double the sum, compared with the result given by the approximation.

	Real time to double (years)	Approximation (years)
x = 5%	14.2	13.8
x = 10%	7.3	6.9
x = 15%	4.9	4.6
x = 20%	3.8	3.4

So even for interest rates up to 20%, the approximation gives

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a result correct to within nearly a 10% level of error. It always understates, and is of course most accurate when x is small: if one wanted to have a system which was most accurate around, say, 10%, the number 73 could be used instead.

Thank you very much

MT.

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